Estimating Discount for Lack of Liquidity Using the Exogenous Spread Approach

A Short Actuarial Valuation

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A Stochastic Spread as a Valuation Discount

What happens when the bid–ask spread itself is uncertain? The exogenous spread approach extends the constant spread method by allowing the spread to vary stochastically while assuming that the investor's own trade does not affect it.

Let the spread have mean μ_S and standard deviation σ_S. A liquidity-adjusted spread can be represented as

$$S^* = \mu_S + z'_\alpha \sigma_S,$$

where z'_α is a confidence parameter for spread uncertainty. The one-sided liquidity cost is then

$$LC = 0.5 \times S^* \times V.$$

Expressing this cost as a percentage of the liquid reference value V gives

$$\boxed{DLOL_{\text{Exogenous}} = 0.5\left(\mu_S + z'_\alpha \sigma_S\right).}$$

The method therefore adds a volatility allowance to the mean spread before applying the same one-sided half-spread logic used in the constant spread approach. Dowd (2005) [1] presents the exogenous spread liquidity framework; the expression of its isolated liquidity component as a DLOL follows the valuation framing in Polanitzer (2026) [3].

Approach Assumptions

The illustration adopts six assumptions:

1. The asset has a bid–ask spread that can be described by a probability distribution.
2. The spread is stochastic rather than fixed.
3. The spread distribution is summarized here by its mean μ_S and standard deviation σ_S.
4. The investor is assumed to sell an existing position, so the adjustment uses one-half of the liquidity-adjusted spread rather than a full round trip.
5. The investor's own trade does not affect the spread; liquidity is exogenous to the transaction.
6. No separate adjustment is made for endogenous price impact, market depth, execution delay, commissions, taxes or other transaction costs.

The fifth assumption is the defining feature of exogenous liquidity. The spread may change, but those changes are attributed to market conditions rather than to the size or direction of the investor's own order. [1]

The confidence parameter z'_α is a spread-risk parameter and is distinct from any parameter used to model market returns. Dowd notes that it may be calibrated; Bangia et al. (1999) [2] use a value of 3 in their liquidity-risk framework.

The Exogenous Spread Formula

Dowd's exogenous spread liquidity adjustment can be written as

$$LC = 0.5 \times V \times (\mu_S + z'_\alpha \sigma_S).$$

Dividing the liquidity cost by the liquid reference value gives the DLOL directly:

$$DLOL_{\text{Exogenous}} = \frac{LC}{V} = 0.5\,(\mu_S + z'_\alpha \sigma_S).$$

Definitions

V is the liquid reference value of the asset or position. LC is the dollar liquidity cost.

μ_S is the mean relative bid–ask spread and σ_S is the standard deviation of that spread. Both are entered as decimals.

z'_α is the confidence parameter applied to spread uncertainty. The resulting quantity

$$S^* = \mu_S + z'_\alpha \sigma_S$$

may be interpreted as the spread level used for the liquidity adjustment.

If $\sigma_S = 0$, then $S^* = \mu_S$ and the formula reduces to

$$DLOL_{\text{Exogenous}} = 0.5\mu_S,$$

which is the constant spread result when $\mu_S = Spread$.

Numerical Illustration

Consider the ABC stock example supplied with the exogenous spread approach. The current stock value is \$100. The mean bid–ask spread is 2%, the standard deviation of the spread is 1%, and the spread confidence parameter is $z'_\alpha = 3$.

The liquidity-adjusted spread is

$$S^* = 0.02 + 3(0.01) = 0.05 = 5\%.$$

The one-sided liquidity cost on a \$100 position is therefore

$$LC = 0.5 \times 100 \times 0.05 = \$2.50.$$

Thus,

$$DLOL_{\text{Exogenous}} = \frac{2.50}{100} = 0.025 = 2.5\%.$$

Equivalently,

$$\boxed{DLOL_{\text{Exogenous}} = 0.5\,(2\% + 3 \times 1\%) = 2.5\%.}$$

The liquidity-adjusted value implied by this exogenous-spread DLOL is \$97.50 for a position whose liquid reference value is \$100.00.

Constant versus Exogenous Spread

The exogenous approach makes the effect of spread uncertainty explicit. If ABC were valued using only its mean spread of 2% as a constant spread, the estimated discount would be

$$DLOL_{\text{Constant}} = 0.5 \times 2\% = 1.0\%.$$

Allowing the spread to be stochastic gives

$$DLOL_{\text{Exogenous}} = 0.5\,(2\% + 3 \times 1\%) = 2.5\%.$$

Hence,

$$\boxed{1.0\% \quad \longrightarrow \quad 2.5\%}$$

when the volatility allowance is added to the same mean spread.

The difference is

$$DLOL_{\text{Exogenous}} - DLOL_{\text{Constant}} = 0.5 z'_{\alpha} \sigma_S = 1.5\%.$$

This incremental discount is the valuation effect of allowing the quoted spread to be uncertain under the stated calibration.

Importantly, the comparison is between two liquidity models. Return-distribution and market-risk components are outside the DLOL calculation; only the liquidity spread and its uncertainty are retained.

Python Implementation

The following implementation reports the liquidity-adjusted spread, dollar liquidity cost and DLOL. The mean spread and spread volatility are entered as decimals.

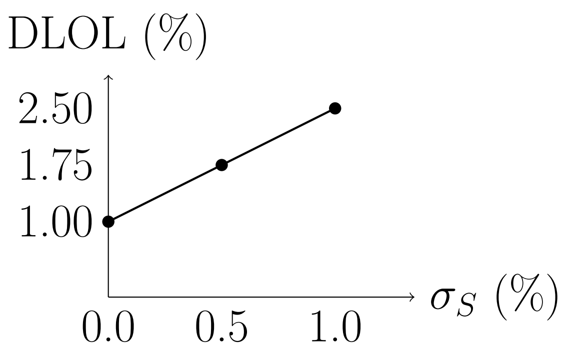
```
def DLOLExogenous(value, mean_spread,
                  spread_vol, z_spread):
    if value <= 0:
        raise ValueError("value must be positive")
    if mean_spread < 0 or spread_vol < 0:
        raise ValueError("spread inputs must be nonnegative")

    adjusted_spread = mean_spread + z_spread*spread_vol
    dlol = 0.5 * adjusted_spread
    liquidity_cost = value * dlol

    return adjusted_spread, liquidity_cost, dlol

s, lc, dlol = DLOLExogenous(
    100, 0.02, 0.01, 3)

print(f"Adjusted spread: 100*s:.3f%")
print(f"Liquidity cost: lc:.2f")
print(f"DLOL: 100*dlol:.3f%")
```

The output is an adjusted spread of 5.000%, a liquidity cost of 2.50, and a DLOL of 2.500%. Setting `spread_vol` to zero reduces the function to the constant spread result based on the mean spread.

Sensitivity and Limitations

Holding $\mu_S = 2\%$ and $z'_\alpha = 3$ fixed, the effect of spread volatility is:

σ_S	Adjusted spread S^*	$DLOL_{\text{Exogenous}}$
0.0%	2.0%	1.00%
0.5%	3.5%	1.75%
1.0%	5.0%	2.50%



The approach requires an empirically defensible spread distribution and confidence parameter. A normal mean–standard-deviation summary may be inadequate when spread data are skewed, heavy-tailed or regime-dependent. Dowd notes that other distributions and Monte Carlo simulation may be used. [1]

The method also remains exogenous: it does not capture the widening of the spread caused by the investor's own trade. For large positions or thin markets, an endogenous liquidity model may be required.

References

[1] Dowd, K. (2005). *Measuring Market Risk*, 2nd ed., Chapter 14: "Estimating Liquidity Risks," pp. 309–320. John Wiley & Sons.

[2] Bangia, A., Diebold, F. X., Schuermann, T., & Stroughair, J. (1999). "Liquidity on the Outside." *Risk*, 12 (June), 68–73.

[3] Polanitzer, R. (2026). "It's Not a DLOM, It's a DLOL!" *The Valuator*, August 3, 2026. (In Hebrew).

Source and implementation note

This article isolates the exogenous-spread liquidity component of Dowd's framework [1] and expresses that component as a DLOL in the valuation sense discussed by Polanitzer [3]. Broader market-risk components are intentionally excluded from the DLOL calculation.

Dowd's exogenous approach treats the spread as stochastic but assumes that a single trader's transactions do not affect it. The liquidity adjustment uses the spread mean plus a confidence parameter times spread volatility. Dowd cites Bangia et al. (1999) [2] for the use of $z'_\alpha = 3$ in this framework.

The numerical illustration retains the English ABC example: a \$100 stock value, mean spread of 2%, spread standard deviation of 1%, and $z'_\alpha = 3$. These inputs imply an adjusted spread of 5%, a one-sided liquidity cost of \$2.50, and a DLOL of 2.5%.

Computerized calculations retain full precision; displayed figures may show immaterial rounding differences.