Estimating Discount for Lack of Liquidity Using the Endogenous Price Approach

A Short Actuarial Valuation

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Price Impact as a Valuation Discount

What happens when the act of selling a position moves the market price itself? The endogenous price approach treats liquidity cost as a price-impact problem rather than only as a bid–ask spread problem.

The constant and exogenous spread approaches assume that market prices do not change in response to the investor's trade. The endogenous approach relaxes that assumption. A sufficiently large sale may place downward pressure on price, creating an additional liquidity loss. Dowd (2005) [1] represents this response through price elasticity:

$$E = \frac{\Delta P/P}{\Delta N/N},$$

where $\Delta N/N$ is the size of the trade relative to the market.

For a sale, E is generally negative and $\Delta N/N$ is positive. Hence the proportional price effect is negative:

$$\frac{\Delta P}{P} = E\frac{\Delta N}{N} < 0.$$

Isolating this loss as a valuation discount gives

$$\boxed{DLOL_{\text{Endogenous}} = -E\frac{\Delta N}{N}.}$$

The DLOL is therefore the absolute proportional price decline attributable to the investor's own transaction.

Approach Assumptions

The illustration adopts six assumptions:

1. The position is sold into a market whose price can respond to trading activity.
2. The sale represents a positive fraction $\Delta N/N$ of the relevant market quantity.
3. Price impact is summarized by a constant elasticity parameter E over the transaction considered.
4. For a sale, the price response is downward, so $E < 0$ and $\Delta N/N > 0$.
5. The endogenous discount isolates market impact caused by the investor's own trade.
6. Bid–ask spreads, spread uncertainty, commissions, taxes, execution delay and other transaction costs are excluded from this isolated adjustment.

The sixth assumption is important. Dowd [1] notes that the endogenous price approach is narrow because it ignores bid–ask spreads and transaction costs. That limitation is useful here because it allows the endogenous price effect to be isolated as a separate DLOL component.

The approach is especially relevant when a position is large relative to market depth. As the transaction becomes larger, or as prices become more responsive to trades, the implied discount increases.

The Endogenous Price Formula

Price elasticity is defined as

$$E = \frac{\Delta P/P}{\Delta N/N}.$$

Rearranging gives the proportional price response:

$$\frac{\Delta P}{P} = E\frac{\Delta N}{N}.$$

Because a liquidity discount is stated as a positive reduction from the liquid reference value, the endogenous DLOL is

$$\boxed{DLOL_{\text{Endogenous}} = -\frac{\Delta P}{P} = -E\frac{\Delta N}{N}.}$$

Definitions

P is the market price before the transaction and ΔP is the price change attributable to the trade.

N represents the relevant market quantity and ΔN is the size of the position to be liquidated. Thus $\Delta N/N$ measures trade size relative to the market.

E is the elasticity of price with respect to the proportion of the market traded. For a sale with downward price pressure, $E < 0$.

If either $E = 0$ or $\Delta N/N = 0$, the endogenous price discount is zero. The formula contains no market-return volatility or VaR parameter because only the liquidity-related price impact is retained.

Numerical Illustration

Consider the English endogenous-price example. A trader holds a position equal to 10% of the size of the market:

$$\frac{\Delta N}{N} = 0.10.$$

The estimated price elasticity is

$$E = -0.40.$$

The proportional price effect of liquidation is therefore

$$\frac{\Delta P}{P} = E\frac{\Delta N}{N} = (-0.40)(0.10) = -0.04.$$

Thus the sale is expected to reduce the market price by 4%. Expressed as a positive valuation discount,

$$DLOL_{\text{Endogenous}} = -(-0.04) = 0.04.$$

Hence,

$$\boxed{DLOL_{\text{Endogenous}} = 4.0\%.}$$

For example, a liquid reference value of \$100 would correspond to \$96 after applying this isolated endogenous-price discount.

What Endogeneity Adds

The endogenous approach measures a different source of liquidity cost from the first two DLOL methods in this series.

The constant spread approach measures the one-sided cost implied by a fixed bid–ask spread:

$$DLOL_{\text{Constant}} = 0.5 \times Spread.$$

The exogenous spread approach allows that spread to be stochastic:

$$DLOL_{\text{Exogenous}} = 0.5\left(\mu_S + z'_\alpha \sigma_S\right).$$

The endogenous price approach instead measures the price movement caused by the liquidation itself:

$$DLOL_{\text{Endogenous}} = -E\frac{\Delta N}{N}.$$

Thus the three methods answer different liquidity questions. A spread can be stable, a spread can fluctuate for market-wide reasons, or the investor's own order can move the price. Dowd [1] notes that endogenous price impact can be combined with other liquidity adjustments in a broader risk framework. In this article, however, the endogenous component is kept separate so that its valuation meaning is transparent and double counting can be avoided.

Python Implementation

The following function computes the proportional price impact and the corresponding endogenous DLOL. Trade size is entered as a fraction of the relevant market.

```
def DLOLEndogenous(elasticity, trade_fraction):
    if trade_fraction < 0:
        raise ValueError("trade_fraction must be nonnegative")

    price_impact = elasticity * trade_fraction
    dlol = -price_impact

    return price_impact, dlol

impact, dlol = DLOLEndogenous(-0.40, 0.10)

print(f"Price impact: 100*impact:.3f%")
print(f"DLOL: 100*dlol:.3f%")
```

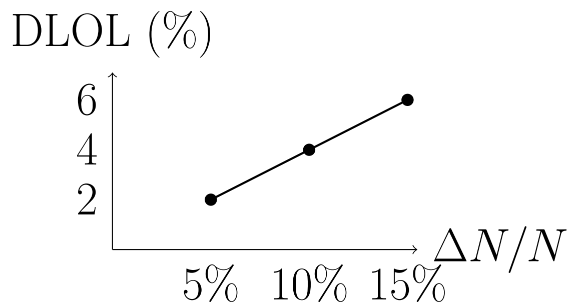
The output is a price impact of -4.000% and a DLOL of 4.000%.

For a sale, the function is economically meaningful when the estimated elasticity is negative. If the input elasticity were positive, the formula would imply a price increase rather than a liquidity discount and would require interpretation rather than mechanical use.

Sensitivity and Limitations

Holding $E = -0.40$ fixed, the effect of transaction size is:

$\Delta N/N$	$\Delta P/P$	$DLOL_{\text{Endogenous}}$
5%	−2.0%	2.0%
10%	−4.0%	4.0%
15%	−6.0%	6.0%



The simple elasticity approach assumes a stable relationship between trade size and price impact. Real market impact may be nonlinear, temporary or permanent, and may depend on order execution strategy, market depth and prevailing conditions. [1]

Its narrow focus is also a limitation: bid–ask spreads and transaction costs are ignored. Jarrow and Subramanian (1997) [2] develop a more sophisticated liquidity framework incorporating both exogenous and endogenous market liquidity, an optimal liquidation horizon and additional liquidity-risk effects.

References

[1] Dowd, K. (2005). *Measuring Market Risk*, 2nd ed., Chapter 14: "Estimating Liquidity Risks," pp. 309–320. John Wiley & Sons.

[2] Jarrow, R. A., & Subramanian, A. (1997). "Mopping Up Liquidity." *Risk*, 10 (December), 170–173.

[3] Polanitzer, R. (2026). "It's Not a DLOM, It's a DLOL!" *The Valuator*, August 3, 2026. (In Hebrew).

Source and implementation note

This article isolates the endogenous price-impact component of Dowd's framework [1] and expresses that component as a DLOL in the valuation sense discussed by Polanitzer [3]. VaR and LVaR are intentionally excluded from the DLOL calculation.

Dowd's endogenous price approach uses elasticity, E, defined as the proportional change in price divided by the proportion of the market traded. For a sale, the usual case is $E < 0$ and $\Delta N/N > 0$. The isolated valuation discount is therefore $-E(\Delta N/N)$.

The numerical illustration retains the English example: a position equal to 10% of the market and elasticity of -0.40. These inputs imply a -4% price impact and an endogenous DLOL of 4%.

Jarrow and Subramanian [2] are cited by Dowd as a more sophisticated liquidity framework that incorporates both exogenous and endogenous liquidity effects and optimal liquidation considerations.

Computerized calculations retain full precision; displayed figures may show immaterial rounding differences.