Estimating Discount for Lack of Liquidity Using the Uniform Probability Space Approach

A Short Actuarial Valuation

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From Size Premium to DLOL

If a published size premium contains several sources of required return, how much of it should be attributed to lack of liquidity when the exact decomposition is unknown? The Uniform Probability Space Approach treats that allocation itself as uncertain.

The economic motivation begins with the established relationship between size and liquidity. Banz (1981) [1] questioned whether size is itself responsible for the size effect or instead proxies for other correlated factors. Amihud and Mendelson (1986) [2] linked higher bid–ask spreads to higher required returns, while later research treats liquidity as a distinct investment characteristic. [4] Polanitzer (2018) [6] proposed the Uniform Probability Space Approach as a practical way to allocate an unknown portion of a size premium to DLOL when no empirical decomposition is available.

A useful analogy comes from fixed income. A yield spread over the risk-free rate may contain more than one component, including compensation related to credit and liquidity. [3] By analogy, a published size premium may be viewed as containing a fundamental size-related risk component and a liquidity-related component:

$$SP = DR + LP.$$

Here the decomposition is not observed directly. The proposed approach therefore models the unknown fraction of the size premium attributable to DLOL as a random allocation. [6]

Approach Assumptions

The illustration adopts six assumptions:

1. The subject asset is marketable but is exposed to a measurable lack of liquidity.
2. A published size premium, SP, is considered relevant to the asset or to a suitable valuation proxy.
3. The size premium may contain both non-liquidity risk compensation and compensation related to liquidity; the exact split is unknown.
4. The liquidity share of the size premium, denoted λ, is bounded between zero and one.
5. In the absence of information supporting one allocation over another, all values of λ between zero and one are assigned equal probability: $\lambda \sim U(0, 1)$.
6. No separate DLOL is added without checking for double counting with other liquidity adjustments already embedded in the valuation.

The approach is deliberately probabilistic rather than econometric. It does not claim that half of every size premium is caused by illiquidity. Instead, it formalizes complete uncertainty about the allocation by using a uniform probability space. The resulting expected allocation is then used as the valuation estimate.

The Uniform Allocation Formula

Let SP denote the relevant size premium and define the liquidity allocation ratio as

$$\lambda = \frac{DLOL}{SP}.$$

The core assumption is

$$\lambda \sim U(0,1).$$

For a continuous uniform distribution on $[0,1]$,

$$E(\lambda) = \frac{0+1}{2} = 0.5.$$

Since

$$DLOL = \lambda\, SP,$$

the expected DLOL is

$$\boxed{E(DLOL) = E(\lambda)SP = 0.5\, SP.}$$

If the liquid reference value is V, the expected dollar discount is

$$E(LD) = V \times E(DLOL),$$

and the expected liquidity-adjusted value is

$$E(V_{\text{adj}}) = V\big(1 - E(DLOL)\big).$$

Definitions

SP is the relevant size premium. λ is the unknown fraction of that premium attributed to DLOL. V is the value before the liquidity adjustment and LD is the dollar liquidity discount.

Numerical Illustration

Consider a marketable but illiquid asset for which the relevant published size premium is

$$SP = 10\%.$$

Under the Uniform Probability Space Approach,

$$E(\lambda) = 0.5.$$

Therefore,

$$E(DLOL) = 0.5 \times 10\% = 5\%.$$

Hence,

$$\boxed{DLOL_{\text{Uniform}} = 5\%.}$$

For a liquid reference value of 100,

$$E(LD) = 100 \times 0.05 = 5,$$

and

$$E(V_{\text{adj}}) = 100(1 - 0.05) = 95.$$

The result should be interpreted as an expected allocation under complete uncertainty about the liquidity share of the size premium. It is not an empirical estimate that 50% of the size premium is known to arise from illiquidity.

Interpreting the Uniform Allocation

The key object is not the size premium itself, but the unknown allocation ratio

$$\lambda = \frac{DLOL}{SP}.$$

If $\lambda = 0$, none of the size premium is attributed to liquidity. If $\lambda = 1$, the entire size premium is attributed to liquidity. Values between zero and one represent intermediate allocations.

Assigning $\lambda \sim U(0,1)$ says that, without additional information, no allocation in this interval is preferred ex ante. The expected value is therefore 0.5. This replaces an arbitrary point allocation—for example 10%, 20% or 90% of the size premium—with the mean of an explicitly stated probability model.

The expected-value estimate therefore follows directly from the stated probability model:

$$E(DLOL_{\text{Uniform}}) = 0.5\, SP.$$

The economic interpretation is deliberately narrow. The method does not attempt to identify the true liquidity component ex post. It asks what allocation is appropriate when the liquidity share is known only to lie between zero and one and no point within that interval is preferred. Under that information set, the midpoint is the expected allocation.

Python Implementation

The following implementation reports the expected DLOL, dollar liquidity discount and adjusted value under a uniform allocation on $[0, 1]$. The size premium is entered as a decimal.

```
def DLOLUniformProbability(value, size_premium):
    if value <= 0:
        raise ValueError("value must be positive")
    if size_premium < 0:
        raise ValueError("size premium must be nonnegative")

    expected_lambda = 0.5
    dlol = expected_lambda * size_premium
    discount = value * dlol
    adjusted_value = value - discount

    return dlol, discount, adjusted_value

dlol, discount, adjusted = DLOLUniformProbability(
    100, 0.10)

print(f"DLOL: 100*dlol:.2f%")
print(f"Liquidity discount: discount:.2f")
print(f"Adjusted value: adjusted:.2f")
```

The output is a DLOL of 5.00%, a liquidity discount of 5.00 on a 100 reference value, and an adjusted value of 95.00. The function implements the expected-value estimator only; it does not estimate the size premium itself.

Sensitivity and Limitations

Because $E(DLOL) = 0.5SP$, the estimate moves linearly with the selected size premium:

Size premium, SP	Expected DLOL
4%	2%
6%	3%
8%	4%
10%	5%
12%	6%

The table illustrates the central sensitivity result: every one-percentage-point increase in the selected size premium increases the expected DLOL by one-half of a percentage point. The relationship is mechanical because the expected allocation ratio is fixed at 0.5 under the uniform assumption.

The principal limitation is the uniform assumption itself. Equal probability over all allocations from zero to one is a modeling convention for insufficient information, not an empirical finding. If evidence supports a narrower range, an asymmetric distribution or a directly estimated liquidity share, that information should replace the uniform assumption.

The method also depends on selecting an appropriate size premium and on avoiding double counting. It is intended as a DLOL estimator for a marketable but illiquid asset, not as a DLOM estimator for a nonmarketable interest.

References

[1] Banz, R. W. (1981). “The Relationship Between Return and Market Value of Common Stocks.” *Journal of Financial Economics*, 9(1), 3–18.

[2] Amihud, Y., & Mendelson, H. (1986). “Asset Pricing and the Bid–Ask Spread.” *Journal of Financial Economics*, 17(2), 223–249.

[3] Duffie, D., & Singleton, K. J. (1999). “Modeling Term Structures of Defaultable Bonds.” *The Review of Financial Studies*, 12(4), 687–720.

[4] Ibbotson, R. G., Chen, Z., Kim, D. Y.-J., & Hu, W. Y. (2013). “Liquidity as an Investment Style.” *Financial Analysts Journal*, 69(3), 30–44.

[5] Duff & Phelps. (2015). *2015 Valuation Handbook: Guide to Cost of Capital*. John Wiley & Sons.

[6] Polanitzer, R. (2018). “The Relationship Between the Size Premium and Liquidity: A New Technique for Estimating the Discount for Lack of Liquidity (DLOL).” Intrinsic Value, October 2018. (In Hebrew).

Source and implementation note

The Uniform Probability Space Approach presented here follows Polanitzer (2018) [6], where the unknown share of a relevant size premium attributable to DLOL is modeled on a uniform probability space. The fixed-income decomposition is used only as a motivating analogy for the possibility that an observed required-return premium contains more than one economic component. [3]

The numerical illustration is hypothetical and uses a 10% size premium for transparency. The assumption $\lambda \sim U(0,1)$ and the resulting estimator $E(DLOL) = 0.5SP$ are the defining assumptions of the approach presented in this note; they are not empirical conclusions of references [1–5].

Computerized calculations retain full precision; displayed figures may show immaterial rounding differences.